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14. ABSTRACT

We have studied the physics of the relative state of quantum detectors that move with respect to each other and their sources, that are accelerated with respect to the source, and that are placed within the gravitational field of a black hole. We outline the general theory of how the entanglement of polarized photons changes under relativistic Lorentz transformations, and have studied quantum information transmission in the presence of a black hole. A description of the accretion of photons by black holes within curved-space quantum field theory has revealed that information is not lost as the photons are absorbed by the black hole because the process of stimulated emission of radiation guarantees that information always stays outside of

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Report Title

Final Report: Relativistic Quantum Information Theory

ABSTRACT

We have studied the physics of the relative state of quantum detectors that move with respect to each other and their sources, that are accelerated with respect to the source, and that are placed within the gravitational field of a black hole. We outline the general theory of how the entanglement of polarized photons changes under relativistic Lorentz transformations, and have studied quantum information transmission in the presence of a black hole. A description of the accretion of photons by black holes within curved-space quantum field theory has revealed that information is not lost as the photons are absorbed by the black hole because the process of stimulated emission of radiation guarantees that information always stays outside of the event horizon, thus solving the black hole information paradox. We also show that stimulated emission turns a black hole into a nearly optimal quantum cloning device, and calculate the cloning fidelity as a function of the black hole absorption coefficient. Finally, we study stimulated emission for accelerated grey bodies in Rindler space, and formulate a framework for consecutive measurements of the same quantum system that allows for a description of the causal dynamics of quantum systems without reference to a time variable.

List of papers submitted or published that acknowledge ARO support during this reporting period. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

R.M. Gingrich, A.J. Bergou, C. Adami, "Entangled light in moving frames", Phys. Rev. A 68 (2003) 042102

H. Lee, U. Yurtsever, P. Kok, G.H. Hockney, C. Adami, S.L. Braunstein, and J.P. Dowling, "Towards photostatistics from photon-number discriminating detectors". J. Mod. Optics 15 (2004) 1517--1528.

D.R. Mitchell, C. Adami, W. Lue, and C.P. Williams, "Random matrix model of quantum computing". Phys. Rev. A 71 (2005) 052324.

Number of Papers published in peer-reviewed journals: 3.00

(b) Papers published in non-peer-reviewed journals or in conference proceedings (N/A for none)

Number of Papers published in non peer-reviewed journals: 0.00

(c) Presentations

"Black Holes Conserve Information in Curved-Space Quantum Field Theory"
presented at QCPR 2004, Orlando, FL, August 19, 2004

"Black Holes Conserve Information in Curved-Space Quantum Field Theory"
presented at 35th Winter Colloquium Physics of Quantum Electronics, Snowbird, UT, January 2, 2005

Number of Presentations: 2.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

C. Adami, "Toward a Fully Relativistic Theory of Quantum Information", Proc. 2003 AMOS Technical Conference, Wailea, Maui (2003)

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts): 1

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts): 0

(d) Manuscripts

C. Adami, The physics of information, eprint quant-ph/0405005 (2004)

C. Adami and G.L. Ver Steeg, Black holes conserve information in curved-space quantum field theory, eprint gr-qc/0407090 (2004)

C. Adami and G.L. Ver Steeg, Black holes are almost optimal quantum cloners, eprint quant-ph/0601065 (2006)

G.L. Ver Steeg and C. Adami, Thermodynamics of accelerated grey bodies. Unpublished, to be submitted

C. Adami, Quantum mechanics of consecutive measurements, to be submitted to Phys. Rev. Lett.

Number of Manuscripts: 5.00

Number of Inventions:

Graduate Students

NAME	PERCENT_SUPPORTED
Greg L. VerSteeg	0.50
Robert Forster	0.25
Jeffrey Edlund	0.25
FTE Equivalent:	**1.00**
Total Number:	**3**

Names of Post Doctorates

NAME	PERCENT_SUPPORTED
Claus Wilke	0.50
FTE Equivalent:	**0.50**
Total Number:	**1**

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics
This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period:	0.00
The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:......	0.00
The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:......	0.00
Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):......	0.00
Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:......	0.00
The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense	0.00
The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:	0.00

Names of Personnel receiving masters degrees

NAME

Total Number:

Names of personnel receiving PHDs

NAME

Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Sub Contractors (DD882)

Inventions (DD882)

Final Report
Relativistic Quantum Information Theory
Army Research Office Grant # DAAD-0301-0207

Christoph Adami

November 16, 2007

1 Foreword

The stated goal of the research program in "Relativistic quantum information theory" was to explore and develop the theory of information into realms that were previously untouched: the realm of moving and accelerated observers. It might at first sight seem odd that this step had not yet been taken: after all the special and general theory of relativity are now respectively a hundred and eighty years old. However, Shannon's theory of information was developed by an engineer and not a physicist, and has, by and large, not enjoyed the attention of physicists until the advent of quantum computation, which necessitated the extension of Shannon's theory to the quantum regime. Still, while some isolated efforts to extend Shannon's theory to moving and accelerated observers occurred (see for example the calculation of the capacity of an information transmission channel including relativistic effects from 1981 [1]), the general subject remained mostly unexplored. This state of affair began to change when the first papers appeared that examined the concept of quantum (that is, von Neumann) entropy with respect to moving detectors. The pioneering work of Peres and students [2, 3] started the quest to understand the concept of information from the much more general point of view of the relative state of detectors.

2 Statement of the problem

Many of the novel applications of quantum information require particles in an entangled state to be shared over long distances, or to be sent from one party to another one in a different location. This requires a deep understanding of the behavior of entanglement under any kind of motion to which the physical system is subjected, which can be provided by a rigorous application of relativistic quantum mechanics. Even in the non-relativistic case, entanglement has proven to be a delicate property, that can be decreased and even

destroyed by the uncontrollable interactions of the system with its environment. Therefore, a clear understanding of entanglement in a relativistic setting might prove crucial to the viability of many applications of quantum information theory.

Quantum effects also play a role when accelerated observers, or equivalently gravitational effects, come into play. That something interesting must happen to entropies in non-inertial frames is immediately clear from the Unruh effect [4, 5, 6]. The Unruh effect is perhaps the most important clue to our understanding of quantum field theory in curved space time, which is still quite incomplete. Accelerated observers perceive a vacuum quite different from that apparent to a non-accelerated observer: they find themselves surrounded by thermal photons of temperature $T_U = \frac{\hbar a}{2\pi c}$ (the Davies-Unruh temperature), where a is the observer's acceleration, and c is the speed of light. If we were to calculate the entropy of a particle in the inertial vs. the non-inertial frame, the absence or presence of the Unruh radiation implies that they would be different. In other words, standard thermodynamic or von Neumann entropies do not transform covariantly under general co-ordinate transformations, that is, they are not scalars. A consistent analysis of quantum information in accelerated observes and curved space time should allow us to resolve paradoxes that have plagued physics for over thirty years.

3 Summary of the most important results

The step we took towards the goal of a fully relativistic theory of quantum information under the present grant built on earlier work by our group, where we studied the entanglement of two massive spin-1/2 particles under Lorentz boosts [7]. Under this grant, we began by applying this theory to massless spin-1 particles, that is, photons, because these are easier to manipulate, and are more likely to be used in a quantum communication setting [8]. We found the transformation law for helicity states and showed that, while that law is frequency independent, a Lorentz transformation on a momentum-helicity eigenstate produces a momentum-dependent phase. This phase leads to changes in the reduced polarization density matrix, such that entanglement is either decreased or increased, depending on the boost direction, the rapidity, and the spread of the beam. Boosting a detector–even at an angle–towards the beams increases this entanglement because the momentum distribution is shrunk by the boost. The type of entangled beams that we have investigated are idealizations of realistic states that can be created using parametric down-conversion. In principle, therefore, the effects discussed here should become relevant as soon as linear-optics based quantum technology is created that is placed on systems that move with respect to a detector (or when the detector moves with respect to such a system).

After this work, we set our eyes on understanding the problem of classical and quantum information in curved space time, that is, an extension of the concepts of entropy and information towards general relativity. Historically, there appeared to be an enormous

need for such an analysis, as it was generally believed that information is destroyed by black holes [9], in a process that cannot be accounted for with our known physical laws (because probability would not be conserved in such a process). From our point of view, this conclusion had to be based on an erroneous understanding of information in curved space, and we set out to understand the information transmission capacity of a black hole channel, by studying how elementary particles that were used to encode information fared when being absorbed by a black hole. What we found was nothing short of astonishing: even though the elementary particles (fermions or bosons) indeed disappear forever behind the event horizon, the information does not disappear with it. Rather, an age-old physical process, namely the stimulated emission of radiation [10], ensures that the information is stripped from the particles and remains outside of the event horizon before the particles are absorbed [11]. A plot of the capacity χ of the quantum black hole channel to transmit classical information (Fig. 1) reveals how this channel is just an ordinary noisy quantum channel, with the noise provided by the famous Hawking radiation. Thus, we show that while the information that is incident on the black hole does not reappear in Hawking radiation–as this radiation is just noise–it does reappear in the form of stimulated radiation. This should have been clear from the outset: Einstein taught us that black bodies absorb radiation, and emit stimulated radiation in response to the absorption as well as spontaneously. And it is known that black holes are in fact black bodies [12].

We subsequently analyzed this problem from the point of view of quantum cloning, because the process of stimulated emission (which was ignored by Hawking and almost all of the authors after) appears to make *copies* of the quantum information outside of the event horizon before absorption [13]. However, it turns out that these are only approximate copies, because the process of *spontaneous* emission of radiation is unavoidable. This spontaneous emission was not ignored by Hawking: in fact it is just what is known as Hawking radiation. We were able to calculate the fidelity of cloning of one quantum state to M approximate clones: $F_{1\to M}$, for black holes with arbitrary reflectivity. For perfectly reflecting black holes (mirrors), the cloning fidelity is that of th optimal cloning machine, whereas the cloning fidleity tends toward the level achievable by optimal state estimation in the limit of perfectly absorbing black holes. Intermediate cases are shown in Fig. 2.

Even though these developments should have galvanized the community of black hole physicists, the opposite happened. The referees of these papers, which we submitted to Physical Review Letters (and the first one later also to Nature Physics), were incredulous. Even though they could not point to any mistakes in the arguments or calculations, they simply stated that the solution to the black hole problem could not be so easy. However, none of the referees turned out to be an expert in both quantum gravity and quantum information at the same time, by their own admission. After a particularly vitriolic report by a senior editor of Physical Review that revealed open hostility and contempt to the authors that "do not have a track record in quantum gravity", we decided to abandon our efforts to get these papers published for the time being, and took an alternative approach, namely to demonstrate that the stimulated emission effect that lies behind the solution to

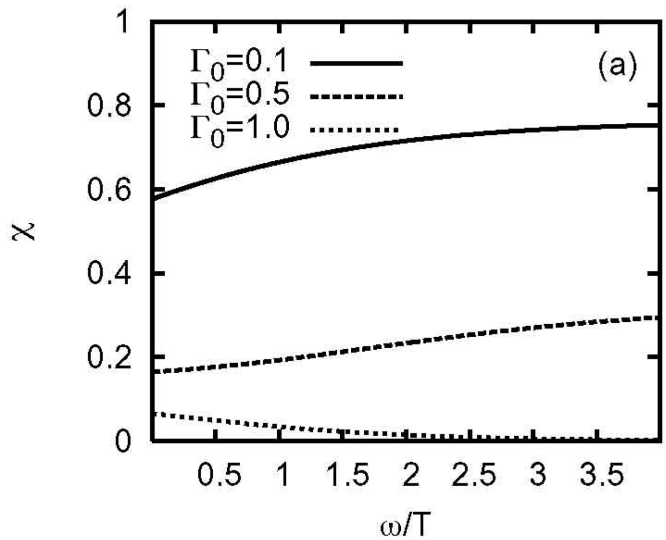

Figure 1: Capacity of the black hole channel to transmit classical information, as a function of the ratio of the frequency of the radiation ω and the temperature of the black hole T, for black holes with three different absorption probabilities: $\Gamma_0 = 1$ refers to a black hole with perfect absorption, while $\Gamma_0 < 1$ refers to black holes that are a little gray. Such gray holes have a higher channel capacity. But the capacity is non-zero throughout, showing that information is not lost in black hole evaporation.

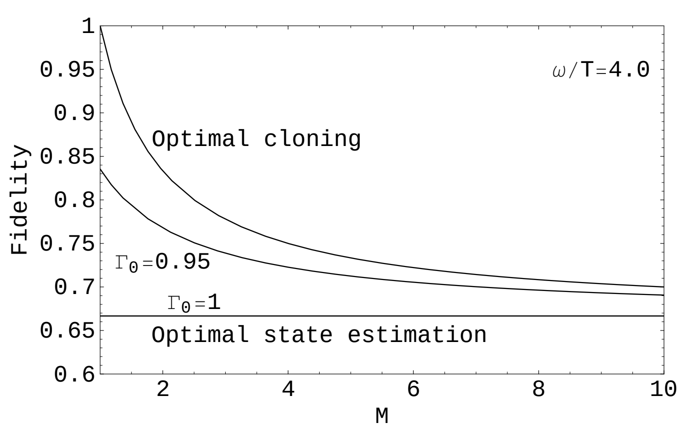

Figure 2: Cloning fidelity $F_{1\to M}$ of the quantum black hole as a function of the number of copies M, for different values of the quantum absorption probability Γ_0 and a fixed $\omega/T = 4$.

the black hole information paradox also occurs in the absence of gravity, for the case of accelerated observers. Our thinking was that if you could show that the effect exists also in flat space time, it would be so much easier for the community to accept the effect, as accelerated observers are usually seen as dual to the curved-space application. A manuscript outlining this approach is in development but has not yet been submitted [14]. We have also another unfinished manuscript that summarizes the physics of information as a theory of the relative state of classical and quantum detectors, that we plan to submit to *Reports on Progress in Physics* when time allows.

Very recently, we started a research program to understand the physics of consecutive quantum measurements from a general point of view, because it is obvious that a covariant description of quantum measurement cannot use a time variable to determine the order of quantum measurements on a physical system. Instead, so-called entropic chains can be used that reflect the causal relationship between detectors, and that reproduce all known quantum results without reference to a time variable. This work is not yet completed, but a preliminary manuscript is available [15].

In summary, we believe that we have made considerable strides towards a fully relativistic theory of classical and quantum information, but that more research is necessary. We hope that additional funding in the future will allow us to finish the work we started.

A List of manuscripts produced under ARO grant # DAAD-0301-0207

All these manuscripts acknowledge funding from the Army Research Office.

A.1 Published in peer reviewed journals

1. R.M. Gingrich, A.J. Bergou, and C. Adami, Entangled light in moving frames, *Phys. Rev.* **A 68** (2003) 042102

2. H. Lee, U. Yurtsever, P. Kok, G.H. Hockney, C. Adami, S.L. Braunstein, and J.P. Dowling, Towards photostatistics from photon-number discriminating detectors. *J. Mod. Optics* **15** (2004) 1517–1528.

3. D.R. Mitchell, C. Adami, W. Lue, and C.P. Williams, Random matrix model of quantum computing, *Phys. Rev.* **A 71** (2005) 052324.

A.2 Submitted but unpublished

1. C. Adami, The physics of information, eprint quant-ph/0405005, May 2004

2. C. Adami and G.L. Ver Steeg, Black holes conserve information in curved-space quantum field theory, eprint gr-qc/0407090, July 2004

3. C. Adami and G.L. Ver Steeg, Black holes are almost optimal quantum cloners, eprint quant-ph/0601065, January 2006

A.3 Manuscripts published in non-peer reviewed proceedings

1. C. Adami, Toward a fully-relativistic quantum theory of information, *Proc. of AMOS Technical Conference*, P. Kervin, ed., Maui, Hawaii, 8-13 September (2003)

A.4 Manuscripts not yet submitted

1. G. L. Ver Steeg and C. Adami, Thermodynamics of accelerated grey bodies (2007)

2. C. Adami, Quantum mechanics of consecutive measurements (2007)

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[15] C. Adami. Quantum mechanics of consecutive measurements. unpublished, October 2007.